

Resistivity of the insulating phase approaching the 2D metal-insulator transition: the effect of spin polarization

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The resistivities of the dilute, strongly-interacting 2D electron systems in the insulating phase of a silicon MOSFET are the same for unpolarized electrons in the absence of magnetic field and for electrons that are fully spin polarized by the presence of an in-plane magnetic field. In both cases the resistivity obeys Efros-Shklovskii variable range hopping $\rho(T) = \rho_0 \exp[(T_{ES}/T)^{1/2}]$, with T_{ES} and $1/\rho_0$ mapping onto each other if one applies a shift of the critical density n_c reported earlier. With and without magnetic field, the parameters T_{ES} and $1/\rho_0 = \sigma_0$ exhibit scaling consistent with critical behavior approaching a metal-insulator transition.

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Two-dimensional (2D) electron systems realized in semiconductor heterostructures and on the surface of doped semiconductor devices such as silicon MOSFETs have been intensively studied for more than half a century [1]. Significant advances in fabrication techniques in recent years have yielded samples with greatly increased electron mobility thereby allowing access to lower electron densities, a regime where the energy of interactions between the electrons is dominant and substantially larger than their kinetic energy. Rather than exhibiting a resistivity that increases logarithmically toward infinity as the temperature is reduced [2] as expected within the theory of localization [3], the resistivity of these strongly-interacting dilute 2D electron systems displays metallic-like behavior at low temperatures and insulating behavior as the electron density is reduced below a material-dependent critical electron density n_c (see references [4–6] for reviews). The apparent metal-insulator transition and metallic phase observed in high-mobility, strongly interacting 2D electron systems is widely regarded as one of the most important unresolved problems in condensed matter physics.

Exceptionally large magnetoresistances have been reported in response to in-plane magnetic field in the vicinity of the critical electron density, n_c . For electron densities $n_s > n_c$ on the just-metallic side of the transition, increasing the parallel magnetic field causes the resistivity to increase by several orders of magnitude at low temperatures and reach saturation at a density-dependent field B_{sat} ; for $B > B_{\text{sat}}$ the temperature dependence of the resistivity is that of an insulator. Measurements have confirmed that the saturation of the sample resistivity corresponds to full spin polarization [7, 8]. Since the parallel magnetic field does not couple to the orbital motion of electrons in sufficiently thin 2D systems, this suggests an important role for the electron spins.

The magnetoresistance has been thoroughly investigated on the metallic side of the transition, while considerably less information has been gathered in the insulating phase. In order to better understand the effect of

spin, we embarked on a detailed comparison of the resistivity of the insulating state arrived at by: (1) reducing the electron density below the critical density n_c in zero field so that the electrons are unpolarized, or (2) applying an in-plane magnetic field beyond the saturation field B_{sat} where the metallic behavior is suppressed and the spins are fully polarized.

It has been established in a number of experiments that the application of in-plane magnetic field causes a shift of the critical density n_c [9–13]. As further detailed below, there have been conflicting reports on the behavior of the resistivity in the insulating phase of low-disorder 2D materials, with some claiming simply-activated behavior and others claiming Efros-Shklovskii variable-range hopping in the presence of a Coulomb gap due to electron interactions [14].

In this paper we report that both in zero field and in the presence of an in-plane magnetic field sufficient to polarize all the carriers, the resistivity obeys Efros-Shklovskii variable-range hopping. Moreover, we demonstrate that the unpolarized insulator and the fully spin-polarized insulator map onto each other if one simply shifts the critical density n_c . This implies that the transport properties of the insulating state are the same in an unpolarized and in a completely spin-polarized system.

Measurements were performed on silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) between 0.25 K and 2 K in an Oxford Heliox He-3 refrigerator in the absence of magnetic field and in a parallel field of 5 T. Similar to those used in Ref. [15], the high-mobility samples used in our studies ($\mu_{\text{peak}} = 3 \times 10^4 \text{ cm}^2/\text{Vs}$) were fabricated in a Hall bar geometry of width 50 μm and distance 120 μm between the central potential probes; the oxide thickness was 150 nm. Contact resistance was minimized by using a split-gate geometry in which thin gaps are introduced in the gate metallization so that a high electron density can be maintained near the contacts independently of the value of the electron density in the main part of the sample. Electron densities were controlled by applying a positive dc gate voltage

relative to the contacts.

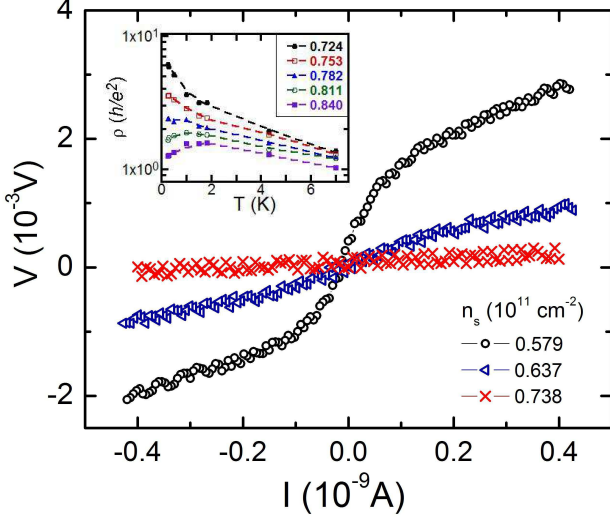


FIG. 1: Nonlinear current-voltage (I-V) characteristics in zero field at several electron densities in the insulating regime; $T = 0.25$ K. As the electron density approaches the critical density from the insulating side, the nonlinearity of I-V gradually fades away [12]. At density $0.738 \times 10^{11} \text{ cm}^{-2}$ which is very close to the critical density, the nonlinearity can only be seen when plotted on an expanded scale. The inset shows the temperature dependence of resistivity at a few electron densities near the critical density in zero field.

As shown in Fig. 1, the voltage is a strongly nonlinear function of the current: it exhibits linear (ohmic) behavior at low currents and bends to a lower slope above a current that depends on electron density and temperature. This is in agreement with numerous past measurements and has variously been attributed to non-ohmic strong electric field Efros-Shklovskii variable range hopping [16], or the depinning of a Wigner Crystal or charge-density wave [17–21]. The resistivity plotted in the next few figures was deduced from the low-current, linear portion of the curves measured for each n_s and T [22].

Figure 2 shows the log of the resistivity measured in zero field plotted against $T^{-1/2}$ for electron densities $n_s < n_c$. For comparison, the inset shows the same data plotted as a function of T^{-1} . Quite clearly, the data obey the Efros-Shklovskii form of variable-range hopping in the temperature range of the measurements (0.25 K to 2 K). Small deviations at the lowest temperature may be due to poor thermal contact of the electron system to the lattice (and thermometer).

Similar data obtained in a parallel field $B_{||} = 5$ T are shown on a semilog plot in Fig. 3, as a function of $T^{-1/2}$ in the main figure and as a function of T^{-1} in the inset. Care was taken to check that $B_{||} = 5$ T is beyond the saturation fields B_{sat} at the densities we used for the measurement, so that all the spins in the sample are fully polarized. The Efros-Shklovskii form of variable-range

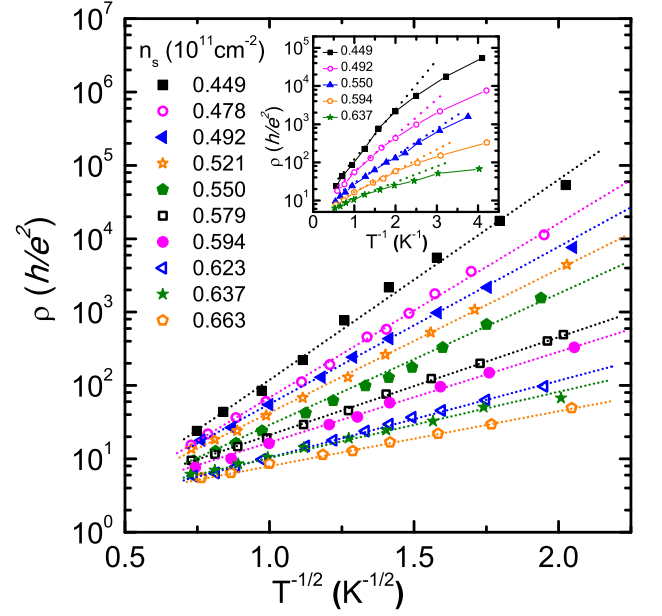


FIG. 2: Resistivity vs $T^{-1/2}$ in zero field for electron densities between 0.449 and $0.663 \times 10^{11} \text{ cm}^{-2}$. For comparison the inset shows the data for several densities plotted vs T^{-1} .

hopping, $\rho(T) = \rho_0 \exp[(T_{\text{ES}}/T)^{1/2}]$ provides an excellent fit to the data in parallel magnetic field.

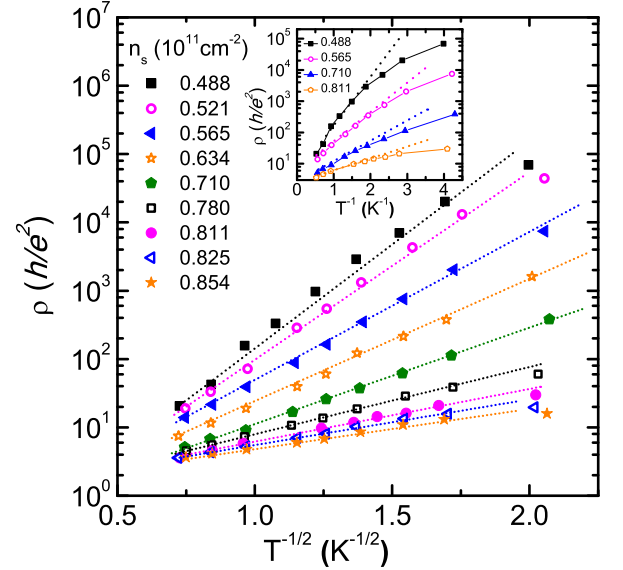


FIG. 3: Resistivity vs $T^{-1/2}$ in 5 T parallel magnetic field for electron densities between 0.488 and $0.854 \times 10^{11} \text{ cm}^{-2}$. The inset shows the data for several densities plotted vs T^{-1} .

In a doped semiconductor at relatively low temperature where there is not enough (thermal) energy to activate electrons across the mobility edge to conducting states in the impurity band, charge transport occurs via thermally activated hopping between localized states,

obeying the expression

$$\rho(T) = \rho_0 \exp[(T_{ES}/T)^\alpha] \quad (1)$$

Here the exponent $\alpha = 1$ for nearest-neighbor hopping, $\alpha = \frac{1}{3}$ for Mott variable-range hopping in two dimensions [23], and $\alpha = \frac{1}{2}$ for Efros-Shklovskii variable-range hopping in the presence of a Coulomb gap in the density of states associated with electron-electron interactions [14, 24].

There have been a number of studies of the resistivity of silicon MOSFETs in the insulating phase. The temperature dependence of the resistivity in zero field was attributed to Efros-Shklovskii variable range hopping with $\alpha = \frac{1}{2}$ down to ≈ 0.3 K by Mertes *et al.* [11] and Jarozynski *et al.* [13] and over a broader range to lower temperatures by Mason *et al.* [22]. The zero-field resistivity was found to obey the same form in a single two-dimensional layer of δ -doped GaAs/Al_xGa_{1-x}As [25]. By contrast, Arrhenius-type activated behavior ($\alpha = 1$) was claimed for silicon in zero field by Shashkin *et al.* [12].

Measurements in the presence of an in-plane magnetic field have yielded various different results: Shashkin *et al.* fitted data for silicon at intermediate temperatures and densities near the transition to the activated form with $\alpha = 1$ [12]; also in silicon, Mertes *et al.* applied a high parallel magnetic field $B_{||} = 10.8$ T and found a larger exponent $\alpha = 0.7$ [26]. For a single two-dimensional layer of δ -doped GaAs/Al_xGa_(1-x)As in parallel fields of 8 T and 6 T, Shlimak *et al.* reported an exponent of 0.8 [25].

As shown in Fig. 2 and Fig. 3 above, the data reported here in zero field and in 5 T parallel field are both consistent with Efros-Shklovskii variable-range hopping, $\rho(T) = \rho_0 \exp[(T_{ES}/T)^{\frac{1}{2}}]$ [27]. The parameters $T_{ES}(0)$ in zero field and $T_{ES}(5T)$ in 5 T parallel field are shown in Fig. 4 as a function of electron density. $T_{ES}(0)$ and $T_{ES}(5T)$ both decrease and gradually approach zero. While $T_{ES}(0)$ extrapolates to zero at critical density n_c in the absence of magnetic field, $T_{ES}(5T)$ obtained in a 5 T parallel field extrapolates to zero at a different electron density which we designate as $n_c(5T) = n_c(B_{sat})$. The inset to Fig. 4 shows that the prefactor ρ_0 increases sharply with increasing electron density.

The critical density n_c in zero field is generally determined as the density at which the temperature derivative of the resistivity, $d\rho/dT$, changes sign; an example of such a "separatrix" between metallic and insulating behavior can be seen in the inset to Fig. 1. The authors of Ref. [12] have shown that in zero field the density where the nonlinearity of current-voltage (I-V) characteristics vanishes yields the same critical density. Using both protocols, the critical density in zero field was estimated to be $n_c = (0.782 \pm 0.014) \times 10^{11} \text{ cm}^{-2}$ for our sample. We note that this is the critical density inferred from thermoelectric power measurements on the same sample [15].

An estimate of the critical density in a 5 T field can be obtained by determining the density at which the non-

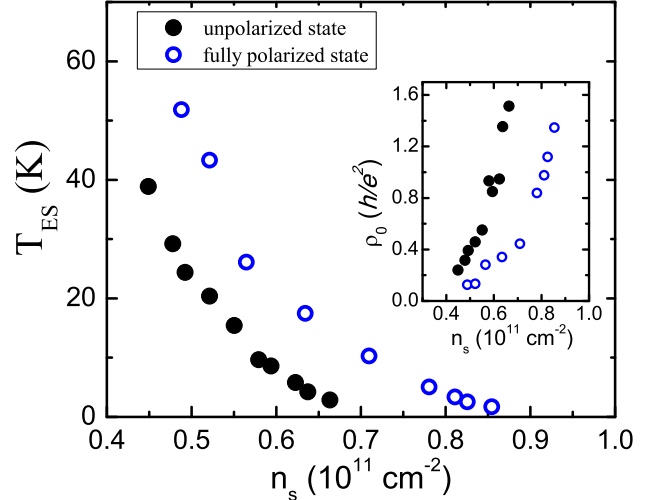


FIG. 4: $T_{ES}(0)$ in zero field and $T_{ES}(5T) = T_{ES}(B_{sat})$ in in-plane field of 5 T vs electron density. The inset shows the parameter ρ_0 as a function of electron density. Closed circles and open circles refer to data in zero field and 5 T, respectively.

linearity in the I-V characteristic vanishes [12]. Using this procedure, an estimate is obtained for the critical electron density at B_{sat} in our sample of $n_c(B_{sat}) = (0.985 \pm 0.014) \times 10^{11} \text{ cm}^{-2}$.

In Efros and Shklovskii's theory of variable-range hopping conduction for strongly interacting electrons, the parameter $T_{ES} \propto \frac{e^2}{\epsilon \xi}$, where ϵ and ξ are the density-dependent dielectric constant and localization length, respectively [14]. Moreover, $\epsilon(n_s) \propto [n_c/(n_c - n_s)]^\zeta$ and $\xi(n_s) \propto [n_c/(n_c - n_s)]^\nu$ as electron density n_s approaches the critical density n_c from the insulating side, so that T_{ES} obeys the critical form:

$$T_{ES} = A[(n_c - n_s)/n_c]^\beta \quad (2)$$

where $\beta = (\zeta + \nu)$.

Using the values estimated for $n_c(0)$ and $n_c(B_{sat})$, we now plot $T_{ES}(0)$ in zero field (filled circles) and $T_{ES}(B_{sat})$ in 5 T parallel field (triangles) as a function of $(n_c - n_s)/n_c$ and $(n_c(B_{sat}) - n_s)/n_c(B_{sat})$, respectively, on a log-log scale in Fig. 5. $T_{ES}(0)$ in the unpolarized state and $T_{ES}(B_{sat})$ in the fully polarized state at 5 T lie on the same curve. Relative to the appropriate critical density for each case, the Efros-Shklovskii parameter $T_{ES} \propto \frac{1}{\epsilon \xi}$ approaches the metallic phase with the same critical exponent, $T_{ES} = A[(n_c - n_s)/n_c]^\beta$ with $\beta = 2.5 \pm 0.1$. Interestingly, the prefactor $1/\rho_0 = \sigma_0$ is also consistent with critical behavior.

Although our data and many past experiments are consistent with the occurrence of a transition, it is noteworthy and puzzling that the value we've obtained for

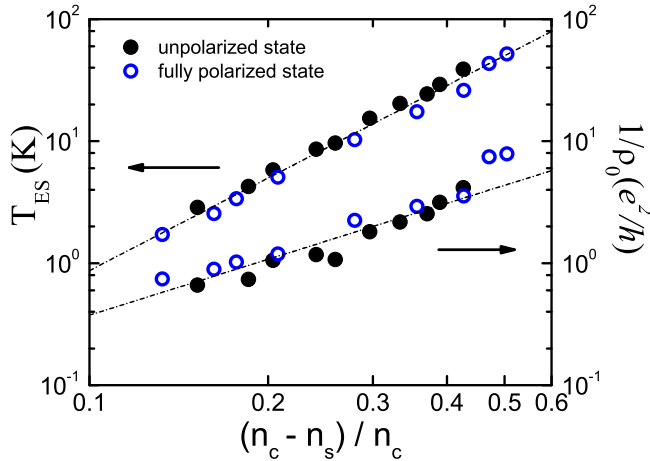


FIG. 5: The parameters T_{ES} and $1/\rho_0 = \sigma_0$ versus $[(n_c - n_s)/n_c]$ on a log-log scale. The filled circles denote the measurements taken in zero field (for unpolarized electrons); the open circles denote measurements taken in a 5T in-plane magnetic field where the electrons are fully spin-polarized. The value of n_c is deduced for each case as described in the text: $n_c(B = 0) = (0.782 \pm 0.014) \times 10^{11} \text{ cm}^{-2}$, $n_c(B_{\text{sat}}) = (0.985 \pm 0.014) \times 10^{11} \text{ cm}^{-2}$.

the critical exponent β is substantially different from the smaller exponents in the vicinity of 1.6 reported in a number of earlier studies [28]. Our analysis is based on data obtained quite far from the transition and entails two parameters, ρ_0 and T_{ES} , while earlier analyses using a single parameter T_0 were based on data obtained near and on both sides of the presumed critical point. A careful examination shows that the portion of our data that is near the transition is consistent with one-parameter scaling and a smaller exponent of $\beta = 1.7$, while the portion of the published data that is further from the transition is can be scaled using two parameters, T_{ES} and ρ_0 with β much closer to our value of 2.5 [29]. Thus, there is no discrepancy between data sets, and the difference may be due to relative closeness to the critical regime.

As suggested in Ref. [15], there may indeed be two quantum critical points in play: n_c driven by disorder, and a disorder-independent universal interaction-driven critical point n_i . They are different in principle, but so close to each other in the low-disorder samples of our studies that they have not been separately identified experimentally. The scaling of T_{ES} and ρ_0 is determined using data obtained in the insulating phase only, and it is by no means clear which (if either) critical point it refers to. This raises the possibility that there may be two closely spaced transitions, one driven by disorder and one driven by interactions. In the range of densities just below the transition where the temperature derivative of the resistivity, $d\rho/dT$, is negative indicating insulating behavior, the resistivity is a very weak function

of temperature so that it cannot be reliably fit to the Efros-Shklovskii form. It is thus possible that there is an intermediate phase in this region that gives rise to different exponents entering and leaving this phase as the density is increased. Additional careful studies to lower temperatures of samples with yet lower disorder in this range of densities would be of great interest.

To summarize, we have measured the resistivity of the dilute, strongly-interacting 2D electron system in a silicon MOSFET in the insulating phase arrived at by (1) reducing the electron density in the absence of magnetic field, which results in no spin polarization (2) applying a 5 Tesla in-plane field which results in complete spin polarization. For both cases, the resistivity obeys Efros-Shklovskii variable-range hopping with parameters T_{ES} and ρ_0 that are consistent with critical behavior approaching a metal-insulator transition. The sole effect of spin polarization is a simple shift of the critical density. The fact that the transport properties of the insulating state are the same in two systems with different spin configurations should be very interesting and worth further theoretical attention.

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